Standards Under Fire: Issues and Options in the Math Wars

Summary of Keynote Session by
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For over 15 years mathematics education has been at the center of discussion and action aimed at reforming curricula, teaching, and assessment in American schools and universities. Prodded by a series of critical national advisory reports and by disappointing results from international comparisons of student mathematics achievement, the National Council of Teachers of Mathematics (NCTM) formulated an agenda for reform in three volumes of professional *Standards*. The National Science Foundation provided extensive funding for mathematics curriculum development projects at all levels and for dozens of large-scale systemic change projects to enhance teacher knowledge and skills and prepare the way for implementation of proposed reforms.

But just as the new curricula, teaching methods, and assessment strategies are being tested in schools and universities across the country and beginning to show promise of reaching the reform objectives, critics have challenged the content goals, pedagogical principles, and assessment practices that are at the heart of the reform agenda. What had seemed to be an overwhelming national consensus on directions for change in mathematics education is facing passionate resistance from dissenting mathematicians, teachers, and concerned citizens. Wide dissemination of the criticisms—through reports in news-media, internet mailings, and debates in the meetings and journals of mathematics professional societies—has shaken public confidence in the reform process. As a consequence there seems a genuine risk that many schools will reject opportunities for much-needed change in mathematics education and continue with comfortable, if demonstrably inadequate, conventional curricula, teaching, and testing practices.

The spirited debates about reform of school and undergraduate mathematics have led some proponents and opponents of change to such angry rhetoric that the controversy is often referred to as the ‘Math Wars.’ In this sort of emotionally charged atmosphere it is very hard to make informed and balanced judgments about the key issues. It's easy to lose sight of the reasons that major reforms were called for in the first place, the rationale and supporting evidence for dominant reform proposals, and the evidence that recent reform initiatives improve the effectiveness of school and university mathematics for most students. Since the critics have gotten most attention in recent public discourse about school mathematics, it seems appropriate to review the situation from a broader perspective—to assess the case for change, the objections of critics, and the evidence supporting each position.

Why Change?

The first thing to keep very clearly in mind as critics resist *Standards*-based proposals for change in school mathematics is the fact that reform initiatives have been prompted by overwhelming evidence that our traditional curricular and teaching practices are not yielding the kind of learning that is both desirable and possible. For almost 30 years, evidence from international comparative studies, our own National Assessment of Educational Progress, and the SAT/ACT college admission testing programs has suggested that, in relation to other countries that are our intellectual and economic competitors,

- Our curricula do not challenge students to learn important topics in depth;
- Our teaching traditions encourage students to acquire routine procedural skills through a passive classroom routine of listening and practicing;
- Our assessment of student knowledge emphasizes multiple choice and short answer responses to low-level tasks.

In response to these challenges, recent reform proposals and emerging curriculum materials have suggested major change in the content, teaching strategies, and assessment practices of school and collegiate mathematics.

New Curriculum Content and Organization

As our curricular traditions have been reconsidered over the past decade, several major changes have seemed in order.

- Emerging calculator and computer technologies have suggested new conceptions of basic skills in arithmetic and algebra—greater emphasis on number/symbol sense and strategies for mental/approximate computation with diminished attention to paper and pencil algorithms for complex calculation.
- Increased emphasis on useful topics from statistics, probability, and discrete mathematics.
- Increased attention to authentic applications of mathematical concepts, skills, and reasoning processes.
- Integration of content strands to form a coherent mathematics curriculum, not a collection of disjoint separate topics.
- Development of mathematical habits of mind.
- Increased exposure of *all students* to significant mathematics throughout the K-12 curriculum.

In broad terms it is hard to argue with any of these curricular proposals that are the agenda of the NCTM *Standards* reform movement. However, the specific curriculum materials developed to implement the reform proposals have provided attractive targets for many critics. The innovative programs vary in their objectives, strategies, and effectiveness, but there are some common themes in the criticisms aimed at the *Standards*-based reform. The critics say:

- Procedural skills in arithmetic and algebra are still essential and they must be learned before conceptual understanding and problem solving are possible.
- If mathematics is always taught through applied problem solving, the essential concepts become lost in the problem contexts. Furthermore, mathematical ideas are of interest to many students as a pure intellectual activity.
- New curricula place too great an emphasis on inductive derivation of concepts, denying the unique role of deductive proof that sets mathematics apart from experimental sciences.

- Integrated curricula are shallow and they lack the rigor of courses in which an entire year is devoted to a single subject.
- Emphasis on mathematics for all inevitably waters the curriculum down and limits the challenges that develop our most talented students.

The reform proposals and criticisms are complex issues on which individual perceptions and professional experiences will suggest a variety of reactions. However, in response to the critics there are several key points to keep in mind. First, calculator and computer technology has changed the way that all but a few theoretical scientists and mathematicians do the arithmetic and algebraic calculations that used to dominate goals for school mathematics. That change is not going to be reversed, so increased emphasis on number/symbol sense and estimation seems just the right direction for change in curriculum goals. Furthermore, there is an emerging body of research evidence suggesting that students can indeed develop solid understanding of arithmetic and algebra without first becoming proficient in routine algorithmic calculation. Second, there is overwhelming evidence from evaluations of new applications-intensive curricula that students find such emphasis motivating and interesting and that the connection of mathematical ideas to concrete problems is a powerful aid in learning and retention of those ideas. Third, the argument that new curricula deny the axiomatic and deductive essence of mathematics is undermined by extensive research showing that students do not acquire such proof sense or skill from traditional curricula. Furthermore, one of the most prominent themes of the NCTM *Standards* and emerging curricula is the principle that students should be consistently required to explain their reasoning in support of any mathematical conclusion. Fourth, the claim that integrated curricula are inevitably superficial is contradicted by the fact that only in the United States is high school mathematics organized in the layer-cake style that supposedly gives depth and rigor to study. Materials emerging from the NSF-supported curriculum projects contain more conceptual depth and complex problem solving than is typical of traditional American textbook programs. Inspection of those materials will also put to rest the notion that production of curricula that can engage all students in substantial mathematics must inevitably fail to challenge the best students.

New Teaching and Learning Strategies

In most curriculum reform movements the focus is on mathematical content and grade placement of topics. However, the NCTM *Standards* argue persuasively for almost equal attention to reform of our traditional patterns of classroom instruction. These recommendations about change in teaching and learning of school mathematics are especially significant at the middle grades, where, one could argue, much of the mathematical content of the standards is fairly familiar material.

The most striking underlying principle of *Standards* recommendations about mathematics teaching is the focus on student thinking. Drawing on constructivist ideas about how students acquire personal understanding and skill in mathematics, the NCTM *Professional Teaching Standards* propose that:

- Mathematical ideas should be developed through student work on interesting and challenging problem-solving tasks, often in problems that have authentic contexts.
- Students should very often collaborate in mathematical problem-solving and explorations, with the teacher acting as stimulant and guide rather than as an expositor.
- Procedural knowledge should be developed on a foundation of conceptual understanding.
- Technology can be a powerful tool in helping students to learn mathematical ideas.
- Discourse about mathematical understandings is a powerful strategy for developing and assessing student understanding.
- Heterogeneous grouping of students will yield greatest overall mathematical achievement by all students.

Most of the recent curriculum development projects have produced materials designed to support classroom implementation of this sort of instruction.

While the reform ideas about teaching might seem convincing beyond challenge, critics have argued against several of the key recommendations about teacher/student roles in the classroom. Their basic claim is that greater success in math will come if we teach children the skills we want them to have (rather than) guide them to discover the skills through exploration. In particular, they assert that:

- Discovery is an inefficient method of learning.
- Teaching requires telling.
- Uncontrolled exploration will lead students to errant conceptions of mathematical ideas and skills.
- Cooperative learning really means that a few diligent and intelligent students do all the work.
- Ability grouping of students is necessary to develop top mathematical talent.
- Typical American teachers don't have strong enough mathematical understanding to handle the open environment of discovery learning, so it's better to have them explain clearly the things they do understand.
- Students should not expect learning, especially learning of mathematics, to be fun. It is hard work.

As with the proposals and criticism about curriculum reform, individual educators will have personal reactions to these issues. However, there is solid justification for most of the reform proposals. First, the suggestion that one must choose either "teaching" or "student discovery" is a misleading characterization of the issues. If we arrange classroom activities in such a way that students learn mathematics, we have taught; if we tell students about mathematics but they don't learn (or remember) it, we have not taught. Second, it is true that there are some aspects of mathematics that students cannot "discover", since they are conventions of the mathematical community. However, those conventions are relatively few in number. Third, the recent reform curriculum projects are producing a body of solid evidence in support of *Standards* proposals for problem-based teaching and learning. Students in the reform curricula develop stronger problem solving skills, they retain what they learn better than students of a show-and-tell-and-practice regimen, and they use the problem contexts as effective guides to transfer of their knowledge.

Fourth, there is solid research evidence supporting the efficacy of cooperative learning. In response to the concern that groups are often "carried" by a few members, one could ask how many students "carry" a class discussion with one teacher and thirty students. Fifth, the concern that loosely guided student explorations will lead to errant conceptions underscores the importance of summary discussions in which discoveries are shared with the teacher and other students. However, to suggest that students get only correct and complete conceptions from teacher exposition is to deny extensive research evidence of student misconceptions that have developed from traditional teaching experiences. The suggestion that teachers don't know enough mathematics to adequately guide exploration and clarification of mathematical ideas is contradicted by extensive evidence that teachers, with appropriate professional development preparation, expand their mathematical insights and do a very capable job in the more open environments. Finally, the issue of heterogeneous versus homogeneous grouping by ability and prior achievement is a long-standing controversial point in American education. There are teachers and (especially) parents with strong convictions about the efficacy of ability grouping for talented students. The research evidence in support of their views is at best mixed, and one could argue that even the most talented students will grow in mathematical understanding and interpersonal skills by interaction with less able students. There is a growing body of evidence from the reform curriculum projects that they work best in classes with a wide range of student ability and other relevant experiences. However, all of those materials have also been used in ability-grouped classes.

Assessment of Student Learning

Reflecting insights from international comparative studies and a desire to refocus school mathematics on conceptual knowledge and problem solving, authors of the NCTM *Standards* also proposed significant change in practice of assessment and evaluation. In particular, they argued that assessment should:

- Be aligned with curriculum and teaching goals and practices;
- Be embedded in everyday classroom instruction;
- Be based on work with authentic tasks;

- Invite students to reflect on their personal learning;
- Honor good mathematical thinking, expression, and answers.

Results of individual and group mathematics testing have important consequences for students and schools, so introduction of alternative testing and grading strategies has raised a variety of concerns. Critics have argued that the new forms of assessment:

- Are not as objective or reliable as the standard practices of traditional testing;
- Overlook the fundamental principle that right answers are the most important thing to be credited.
- Are biased against students with less ability to express themselves in writing.
- Place unreasonable demands on teacher time to score open format examinations.
- Prepare students poorly for the critical college admission and placement tests they will face.

It's probably safe to say that many of these concerns are aimed at abuses of new assessment ideas or unfortunate expressions of the reform agenda. For example, in the interest of encouraging students to give more expansive explanations of their reasoning it is easy to suggest that any writing whatsoever is to be credited or that the process of one's thinking is far more important than whether the assessment problems have actually been solved. On the other hand, defending multiple choice or short-answer tests because they mimic traditional college admission or placement testing avoids important questions about the appropriateness of format and content in those high-stakes tests. When mathematics educators in other countries look at the types of questions and responses that typify American assessments, they can't believe that we'd be satisfied with what we expect.

What Does One Do?

When concerns are raised about new curricula, teaching, and assessment in school mathematics, the discourse is typically more like the adversarial style of American courtrooms than the dispassionate search for 'truth' that one associates with scientific work. Public critics of *Standards*-based mathematics programs often suggest that there are only two choices—radical reform or back-to-basics traditional programs. They frequently express their contrary opinions in harsh rhetorical style. They support their judgments with carefully selected illustrations of errors in specific curriculum materials, often from early draft versions of those materials, and with anecdotal evidence of problems experienced by individual students or teachers. The critics also usually fail to acknowledge that the motivation for the whole reform movement was deep concern about the inadequate effects of long-standing American traditions in curriculum, teaching, and testing.

It is difficult to make an informed judgment about any particular approach to reform. Furthermore, in looking for the program that seems best for an individual school or system it's important to keep in mind the differences among reform curricula themselves. It's not easy to chart a thoughtful course through the conflicting claims and complex options, but there is an emerging body of evaluation data that helps. Unfortunately, the results usually offer conflicting choices that will reflect one's personal and societal values. Evidence generally shows that students will learn what you teach and they won't learn what you don't teach. However, it is my judgment that the choices we make do matter and that holding on to the past is not the answer.

We've been through this kind of turmoil many times in the past. The typical pattern is a *Crisis-Reform-Reaction* cycle in which some military or economic challenge to the country prompts urgent calls to improve education as a solution. Reformers propose and develop new curricula (like the "new math"), teaching methods (like using television or computers), or testing practices (like giving more tests for accountability). But before any of the new proposals can be adequately developed and tested, conservative reaction sets in—typically expressed as concern about experimentation with current students. After a burst of enthusiasm, things tend to settle back into the conventional and apparently unchangeable traditions of American education.

In many important ways school mathematics is, if not broken, then functioning far below what we want. There is indeed a risk that ill-advised changes could make things worse. On the other hand, doing nothing is almost certain to make things worse as our curriculum falls behind the development of mathematics and our teaching methods fail to engage large numbers of students in what we all believe is an enormously useful and attractive subject.

Note: Many of the ideas expressed in the talk for which this paper was written were articulated while I worked with colleagues Harold Schoen, Christian Hirsch, and Arthur Coxford on a paper published in the February 1999 *Phi Delta Kappan.* The arguments are supported by extensive references cited in that paper, however I take personal responsibility for the particular views expressed at the May 1999 Show-Me Conference.

Reference: Schoen, H. L., Fey, J. T., Hirsch, C. R., and Coxford, A. F. (1999). Issues and options in the math wars. *Phi Delta Kappan*, 80 (6), 444-453.