Research Supporting NCTM-Standards-Based Mathematics Education Reform

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There is a rigorous and extensive research base for NCTM-Standards-based reform in mathematics education. A brief sample of that research base, related to several major themes of reform, is included here.

1. Teaching Mathematics for Understanding

"There is a long history of research, going back to the 1940s and the work of William Brownell, on the effects of teaching for meaning and understanding in mathematics. Investigations have consistently shown that an emphasis on teaching for meaning has positive effects on student learning, including better initial learning, greater retention, and an increased likelihood that the ideas will be used in new situations. These results have also been found in studies conducted in high-poverty areas." (Grouws & Cebulla, 2000, p. 13)

"Instructional programs that emphasize conceptual development, with the goal of understanding, can facilitate significant mathematics learning without sacrificing skill proficiency." (Hiebert, 2003, p. 16)

"Students who memorize facts or procedures without understanding often are not sure when and how to use what they know, and such learning is often quite fragile." (Bransford, Brown, & Cocking, 1999, cited in NCTM, 2000, p. 20)

"Students who develop conceptual understanding early perform best on procedural knowledge later." (Grouws & Cebulla, 2000, p. 15)

A Few References:

- Grouws, Douglas A. & Cebulla, Kristin J. (2000). *Improving Student Achievement in Mathematics*. Geneva, Switzerland: International Academy of Education.
- Hiebert, James (2003). What research says about the NCTM Standards. In J. Kilpatrick, W. G. Martin, and D. Schifter (Eds.), *A Research Companion to Principles and Standards for School Mathematics* (pp. 5-23). Reston, VA: National Council of Teachers of Mathematics.
- Bransford, John D., Brown, Ann L., & Cocking, Rodney R. (Eds.) (1999). *How People Learn: Brain, Mind, Experience, and School*. Washington, D.C.: National Academy Press.

- National Council of Teachers of Mathematics (NCTM) (2000). *Principles and Standards for School Mathematics*. Reston, VA: NCTM.
- Madsen, Anne L. & Lanier, Perry (1995). Does conceptually oriented instruction enhance computational competence? *Focus on Learning Problems in Mathematics, Fall Edition, Volume 17, Number 4*, 42-64.

2. Teaching Mathematics Using Problem-Based Instructional Tasks (Teaching Mathematics Through Problem Solving)

"What do the findings from research suggest about the feasibility and efficacy of teaching mathematics through problem solving? The research reviewed herein suggests both the feasibility and efficacy of such approaches." (Stein, Boaler, & Silver, 2003, pp. 255-56)

"Problem solving should be the site in which all of the strands of mathematics proficiency converge." (Kilpatrick, Swafford, & Findell, 2001, p. 421)
(Strands are: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition.)

"Students can learn new skills and concepts while they are working out solutions to problems." (Grouws & Cebulla, 2000, p. 15)

A Few References:

- Stein, Mary Kay; Boaler, Jo; Silver, Edward A. (2003). Teaching mathematics through problem solving: Research perspectives. In H. L. Schoen (Ed.), *Teaching Mathematics Through Problem Solving, Grades 6-12* (pp. 245-256). Reston, VA: National Council of Teachers of Mathematics.
- Kilpatrick, Jeremy; Swafford, Jane; Findell, Bradford (Eds.); Mathematics Learning Study Committee, National Research Council (2001). Conclusions and recommendations. In *Adding It Up: Helping Children Learn Mathematics* (pp. 407-432). Washington, D.C.: The National Academies Press.
- Grouws, Douglas A. & Cebulla, Kristin J. (2000). *Improving Student Achievement in Mathematics*. Geneva, Switzerland: International Academy of Education.
- Hiebert, James & Wearne, Diana (1993). Instructional tasks, classroom discourse, and students' learning in second-grade arithmetic. *American Educational Research Journal, 30*, 393-425.

3. Meaningful Distributed Practice of Concepts, Skills, and Problem Solving

"… practice on computational procedures should be designed to build on and extend understanding." (Kilpatrick, Swafford, & Findell, 2001, p. 423)

"If students are to develop both proficiency and understanding of skills, the most efficient instructional approach is to build understanding into the students' experience from the beginning." (Hiebert, 2003, p. 18)

"What's the most efficient way to allocate practice time? … The straightforward answer that we can draw from research evidence is that distributing study time over several sessions generally leads to better memory of the information than conducting a single study session." (Willingham, 2002)

"Practice should be used with feedback to support all strands of mathematical proficiency and not just procedural fluency." (Kilpatrick, Swafford, & Findell, 2001, p. 423)
(Strands are: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition.)

"If students overpractice procedures before they understand them, it is more difficult to make sense of them later." (Hiebert, 2003, p. 17)

"If students are initially drilled too much on isolated skills, they have a harder time making sense of them later." (Grouws & Cebulla, 2000, p. 16)

A Few References:

- Kilpatrick, Jeremy; Swafford, Jane; Findell, Bradford (Eds.); Mathematics Learning Study Committee, National Research Council (2001). Conclusions and recommendations. In *Adding It Up: Helping Children Learn Mathematics* (pp. 407-432). Washington, D.C.: The National Academies Press.
- Hiebert, James (2003). What research says about the NCTM Standards. In J. Kilpatrick, W. G. Martin, and D. Schifter (Eds.), *A Research Companion to Principles and Standards for School Mathematics* (pp. 5-23). Reston, VA: National Council of Teachers of Mathematics.
- Willingham, Daniel (2002). Allocating student study time: "Massed" versus "distributed" practice. *American Educator, Summer 2002*.
- Grouws, Douglas A. & Cebulla, Kristin J. (2000). *Improving Student Achievement in Mathematics*. Geneva, Switzerland: International Academy of Education.
- Rea, Cornelius P. and Modigliani, Vito (1985). The effect of expanded versus massed practice on the retention of multiplication facts and spelling lists. *Human Learning, Volume 4*, 11-18.
- Pesek, Dolores D., and Kirshner, David (2000). Interference of instrumental instruction in subsequent relational learning. *Journal for Research in Mathematics Education, Volume 31* (*5*), 524-540.

4. Assessment for Learning

Extensive research reviews have concluded that stronger formative assessment (assessment for learning) produces significant learning gains. (Black & Wiliam, 1998a, 1998b)

A Few References:

- Black, P., & Wiliam, D. (1998a). Assessment and classroom learning. *Assessment in Education*, *5*(1): 7-74.
- Black, P., & Wiliam, D. (1998b). Inside the black box: Raising standards through classroom assessment. *Phi Delta Kappan, 80*(2): 139-148. http://www.pdkintl.org/kappan/kbla9810.htm

5. Traditional and Reform (NCTM-Standards-Based) Approaches

"On tests of conceptual understanding and problem solving, students who learn from reform curricula consistently outperform students who learn from traditional curricula by a wide margin. On tests of basic skills, there are generally no significant differences between students who learn from traditional or reform curricula." (Schoenfeld, 2002)

"Students in alternative programs implemented with fidelity for reasonable lengths of time have learned more and learned more deeply than in traditional programs." (Hiebert, 2003, p. 20)

"[There is] considerable evidence that the promises of reform mathematics are real and the fears of the anti-reformers unjustified." (Swafford, 2003)

"The studies reported in this book provide much needed evidence that the new programs work." (Kilpatrick, 2003)

"Presuming that traditional approaches have proven to be successful is ignoring the largest database we have." (Hiebert, 2003, p. 13)

In the traditional approach, "… the teacher demonstrates or leads a discussion on how to solve a sample problem. The aim is to clarify the steps in the procedure so that students will be able to execute the same procedure on their own. [Then] students practice using the procedure by solving problems similar to the sample problem." (Stigler & Hiebert, 1997, p. 18)

A Few References:

- Schoenfeld, Alan H. (2002). Making Mathematics Work for All Children: Issues of Standards, Testing, and Equity. *Educational Researcher* 31, no. 1: 13-25
- Hiebert, James (2003). What research says about the NCTM Standards. In J. Kilpatrick, W. G. Martin, and D. Schifter (Eds.), *A Research Companion to Principles and Standards for School Mathematics* (pp. 5-23). Reston, VA: National Council of Teachers of Mathematics.
- Swafford, J. (2003). Reaction to High School Curriculum Projects Research. In S. Senk and D. Thompson (Eds.), *Standards-Based School Mathematics Curricula: What Are They? What Do Students Learn?* Mahwah, NJ: Lawrence Erlbaum Associates.
- Kilpatrick, J. (2003). What Works? In S. Senk and D. Thompson (Eds.), *Standards-Based School Mathematics Curricula: What Are They? What Do Students Learn?* Mahwah, NJ: Lawrence Erlbaum Associates.
- Stigler, J. W., & Hiebert, J. (1997). Understanding and improving classroom mathematics instruction: An overview of the TIMSS video study. *Phi Delta Kappan*, *79*(1), 14-21.

6. General Research Base for Improving Mathematics Teaching and Learning

Published Research Reviews

- National Center for Improving Student Learning and Achievement in Mathematics and Science (2004)
  Powerful Practices: Research-Based Practices for Teaching and Learning Mathematics and Science
  http://www2.learningpt.org/catalog/item.asp?SessionID=468907629&productID=156
- National Council of Teachers of Mathematics (2003)
  Research Companion to Principles and Standards
  http://my.nctm.org/ebusiness/ProductCatalog/product.aspx?ID=12341
- National Council of Teachers of Mathematics (2003)
  Teaching Mathematics Through Problem Solving, Research Chapter
  http://my.nctm.org/ebusiness/ProductCatalog/product.aspx?ID=12577
- *Standards-Based School Mathematics Curricula: What Are They? What Do Students Learn?* (2003)
  https://www.erlbaum.com/shop/tek9.asp?pg=products&specific=0-8058-4337-X
- National Research Council (2001)
  Adding It Up
  http://www.nap.edu/books/0309069955/html/
- International Bureau of Education (2000)
  Improving Student Achievement in Mathematics
  http://www.ibe.unesco.org/publications/EducationalPracticesSeriesPdf/prac04e.pdf

National Textbook Evaluations and Reviews

- National Research Council, 2004
  On Evaluating Curricular Effectiveness: Judging the Quality of K-12 Mathematics Evaluations
  http://books.nap.edu/catalog/11025.html
- American Association for the Advancement of Science, Review of Algebra Texts, 2000
  http://www.project2061.org/publications/articles/textbook/hsalg/default.htm?jsRedirect&txtRef=
- U.S. Department of Education, Exemplary Programs, 1999
  http://www.ed.gov/PressReleases/10-1999/mathpanel.html

Research Review Panels

- Federal What Works Clearinghouse
  http://w-w-c.org/
- Iowa Content Network
  http://www.state.ia.us/educate/ecese/tqt/tc/prodev/mathematics.html

National and International Tests

- NAEP (http://nces.ed.gov/nationsreportcard/mathematics/)
- SAT and ACT (http://www.collegeboard.com/splash and http://www.act.org/)
- TIMSS (http://nces.ed.gov/timss/)
- PISA (http://nces.ed.gov/Surveys/PISA/)