Straight Talk
about Issues in Mathematics Education

The Basics—More than Knowing Computation Facts and Procedures

The National Council of Teachers of Mathematics believes that all students must learn basic computation facts and know how to compute. However, the skills that today's students need are different from those their parents needed.

Basic facts and computation are still important.
Let there be no mistake. Arithmetic is still an important part of elementary school mathematics. Children need to know how to add, subtract, multiply, and divide. Children need to learn addition and subtraction facts (such as 8 + 6 = 14 or 18 - 9 = 9) and multiplication facts (such as 7 x 6 = 42). Fractions, decimals, and percents must be understood. However, it is just as important that children can apply arithmetic in real-life, and understand basic principles of probability, measurement, statistics, and geometry.

Children need to learn what computation means and how to do it.
Research indicates that by solving meaningful word problems, children can learn what it means to add, subtract, multiply, and divide. At the same time, children learn what numbers mean and how numbers are used. They learn strategies to support their recall of basic facts and to develop their computational skills. For example, a child in first grade may say that 6 + 7 is 13 because 7 is 1 more than 6 and 6 + 6 is 12 (Carpenter et al. 1989; Carpenter and Moser 1983).

Every child should have an accurate method for computing 28 x 7, 37 + 58, 1/2 + 3/4, and 193 ÷ 6. Some children will invent their own procedures for computing, and some children will use more conventional methods they have learned from other children or adults. Regardless of the source, children's computational procedures need to be both efficient and correct (Campbell, Rowan, and Suarez 1998). The development of efficient, correct procedures requires careful instruction that focuses on developing understanding.

Practice is important, but practice without understanding is a waste of time.
Once children understand computational procedures, practice will help them become confident and competent in using them. Research indicates that if children memorize mathematical procedures without understanding, it is difficult for them to go back later and build understanding (Resnick and Omanson 1987; Wearne and Hiebert 1988). When children memorize without understanding, they may confuse methods or forget steps (Kamii and Dominick 1998).

As a general rule, parents can expect the following: By the end of second grade, children should know the basic addition and subtraction facts, and by the end of fourth grade, they should know the multiplication facts. By the end of fifth grade, most children should have proficient and accurate methods for computing with whole numbers. Proficiency with fraction and decimal computation should develop during the middle school years. Parents also need to know that their children should not be held back from learning other mathematics, such as geometry or statistics, because they do not yet know all the basic facts or are not yet proficient with computation. Schools should provide support for children to continue working on basic facts and computation while instruction in other mathematics topics takes place.

Students must be prepared for their future.
Topics such as measurement, statistics, geometry, and algebra are just as basic as computation. These topics provide students the opportunity to apply their skills and to learn more advanced mathematics. Schools today have a responsibility to teach the basics--but the basics of today must include algebra, geometry, statistics, and measurement, as well as arithmetic.

References

Campbell, Patricia F., Thomas E. Rowan, and Anna R. Suarez. "What Criteria for Student-Invented Algorithms?" In *The Teaching and Learning of Algorithms in School Mathematics*, edited by Lorna J. Morrow, pp. 49–55. Reston, Va.: National Council of Teachers of Mathematics, 1998.

Carpenter, Thomas P., Elizabeth Fennema, Penelope L. Peterson, Chi-Pang Chiang, and Megan Loef. "Using Knowledge of Children's Mathematics Thinking in Classroom Teaching: An Experimental Study." *American Educational Research Journal* 26 (1989): 499–531.

Carpenter, Thomas P., and James M. Moser. "The Acquisition of Addition and Subtraction Concepts." In *The Acquisition of Mathematical Concepts and Processes*, edited by Richard Lesh and Marcia Landau, pp. 7–44. New York: Academic Press, 1983.

Kamii, Constance, and Ann Dominick. "The Harmful Effects of Algorithms in Grades 1–4." In *The Teaching and Learning of Algorithms in School Mathematics*, edited by Lorna J. Morrow, pp. 130–140. Reston, Va.: National Council of Teachers of Mathematics, 1998.

Resnick, Lauren B., and Susan F. Omanson. "Learning to Understand Arithmetic." In *Advances in Instructional Psychology*, vol. 3, edited by Robert Glaser, pp. 41–95. Hillsdale, N.J.: Lawrence Erlbaum Associates, 1987.

Wearne, Diana, and James Hiebert. "A Cognitive Approach to Meaningful Mathematics Instruction: Testing a Local Theory Using Decimal Numbers." *Journal for Research in Mathematics Education* 19 (1988): 371–384.

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Campbell, Patricia F., Thomas E. Rowan, and Anna R. Suarez. "What Criteria for Student-Invented Algorithms?" In *The Teaching and Learning of Algorithms in School Mathematics*, edited by Lorna J. Morrow, pp. 49–55. Reston, Va.: National Council of Teachers of Mathematics, 1998.

This article suggests criteria by which student-invented algorithms can be evaluated. The criteria include efficiency, mathematical validity, and generalizability. Examples are given.

Carpenter, Thomas P., Elizabeth Fennema, Penelope L. Peterson, Chi-Pang Chiang, and Megan Loef. "Using Knowledge of Children's Mathematics Thinking in Classroom Teaching: An Experimental Study." *American Educational Research Journal* 26 (1989): 499–531.

A group of first-grade teachers in research classes introduced and developed addition and subtraction through word problems rather than through computation-fact problems. There was no difference in the scores on the Iowa Test of Basic Skills between the research and control classrooms. Children in research classes recalled more number facts and scored higher on a word-problem test than children in control classes.

Carpenter, Thomas P., and James M. Moser. "The Acquisition of Addition and Subtracting Concepts." In *The Acquisition of Mathematical Concepts and Processes*, edited by Richard Lesh and Marcia Landau, pp. 7–44. New York: Academic Press, 1983.

This chapter presents a classification of addition and subtraction word problems ordered by difficulty that can be solved by first-grade children. Included is a detailed examination of the strategies that children use to solve these problems over time, as the children develop more understanding and become more abstract in their thinking.

Charles, Randall, and Joanne Lobato. *Future Basics: Developing Numerical Power.* A Monograph of the National Council of Supervisors of Mathematics. Golden, Colo.: National Council of Supervisors of Mathematics, 1998.

This booklet presents a rationale and recommendations for shifting the emphasis in grades K–7 mathematics programs from pencil-and-paper mastery of algorithms to the development of numerical power, whereby students have a strong knowledge of numerical concepts and ideas and proficiency with numbers that they can use in a wide range of situations.

Henningsen, Marjorie, and Mary Kay Stein. "Mathematical Tasks and Student Cognition: Classroom-Based Factors That Support and Inhibit High-Level Mathematical Thinking and Reasoning." *Journal for Research in Mathematics Education* 28 (November 1997): 524–549.

This study focuses on factors that influence the level of mathematical thinking and reasoning that students use during mathematical tasks. Keeping students engaged in high-level mathematical thinking is essential if they are going to learn the important basics of mathematics.

Kamii, Constance, and Ann Dominick. "The Harmful Effects of Algorithms in Grades 1–4." In *The Teaching and Learning of Algorithms in School Mathematics*, edited by Lorna J. Morrow, pp. 130–140. Reston, Va.: National Council of Teachers of Mathematics, 1998.

This chapter addresses the relationship between numerical reasoning and algorithms. The authors contend that requiring specific algorithms hinders the development of children's numerical reasoning.

Leinhardt, Gaea, Ralph Putnam, and Rosemary A. Hattrup, eds. *Analysis of Arithmetic for Mathematics Teaching*. Hillsdale, N.J.: Lawrence Erlbaum Associates, 1992.

This book evolved from a project between the American Federation of Teachers and the Learning Research and Development Center at the University of Pittsburgh. The chapters summarize research on the learning of major arithmetic ideas.

Morrow, Lorna J., ed. *The Teaching and Learning of Algorithms in School Mathematics*. Reston, Va.: National Council of Teachers of Mathematics, 1998.

The chapters in this book address aspects of mathematical algorithms across grades K–12. Major issues surrounding the place of algorithms in school mathematics are addressed, along with historical topics and specific suggestions for instruction in elementary, middle, and high school.

Resnick, Lauren B., and Susan F. Omanson. "Learning to Understand Arithmetic." In *Advances in Instructional Psychology*, vol. 3, edited by Robert Glaser, pp. 41–95. Hillsdale, N.J.: Lawrence Erlbaum Associates, 1987.

This chapter addresses the relationship between understanding and skill performance in multi-digit subtraction. Two instructional programs were designed for students in grades 4, 5, and 6 who had persistent difficulties with the computation. Once children had acquired a relatively automatic but incorrect routine for calculation, instruction to support understanding was often ineffective for correcting calculation errors.

Sowder, Judith T., and Bonnie P. Schappelle, eds. *Providing a Foundation for Teaching Mathematics in the Middle Grades.* Albany, N.Y.: State University of New York Press, 1995.

This book addresses the mathematical ideas for the middle grades. There are suggestions of many specific instructional activities along with a discussion of what happens in classrooms when those suggestions are implemented.

Wearne, Diana, and James Hiebert. "A Cognitive Approach to Meaningful Mathematics Instruction: Testing a Local Theory Using Decimal Numbers." *Journal for Research in Mathematics Education* 19 (1988): 371–384.

Students in grades 4, 5, and 6 were taught the meaning of decimals and decimal computation. Some students had prior conventional instruction in adding and subtracting decimals. Prior instruction that emphasized rules seemed to interfere with the development of understanding of decimal concepts.